TASKS AND LEARNING PATHS IN ASYMPTOTE AND GEOGEBRA

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Abstract. Automatic tutoring systems allow teachers to design tasks to assist students' learning processes. Here we analyse mathematics automatic feedback tasks built with GeoGebra and Asymptote; each task included a series of decisions about pedagogical and curricular strategies inherent to the tasks proposed, related whit a hypothetical learning trajectory. In this exploratory study, we aim to understand views about ATS by analysing data collected from six future teachers and two secondary students. The results reveal that the analysed tasks and learning graph were adjusted, provoked interest in the students, and there is evidence to have triggered a-didactic and mathematical situations with user engagement. Although automatic feedback GeoGebra tasks can provide several mathematics representations and ways of user interaction, they require more creator domain technology; hence, constructing the learning graph is easier with Asymptote.

Key words: Asymptote, automatic feedback, GeoGebra.

INTRODUCTION

Automatic tutoring systems (ATS), such as Asymptote, and other free software, such as GeoGebra, allow teachers to design tasks to assist their students' learning processes. In general, in these systems, the students can receive certified feedback on their answers and access hints that may allow them to overcome some difficulties. In these systems, the creator of the tasks intends to create a scenario with a teaching objective, idealising a learning path and parameterising the system to adjust it to his initial idea. The process of designing learning tasks in learning situations in face-to-face teaching is complex, but it relies mainly on the interaction between the teacher and students. Hence, the communication established in the classroom allows the adjustment of the task and adaptations to the students. In ATS, the interaction between students and the system cannot be monitored in the same way as in face-to-face teaching, which entails new challenges for the teacher in creating tasks with automatic feedback.

To reflect on the construction of tasks for teaching mathematics with automatic feedback, which can be inserted in ATS, we present the theoretical framework that guided this exploratory study that aims to understand future teachers' and secondary students' views about ATS, considering GeoGebra and Asymptote. Next, we present two tasks, one built in GeoGebra and another on the Asymptote platform, created to provide automatic feedback to users guided by the theoretical considerations presented. We also offer in this study the results of the first application of these tasks to two secondary students, as well as the perception of six future teachers about them. Finally, we discuss the results obtained and establish some final remarks.

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THEORETICAL FRAMEWORK

This paper only analyses the application of ATS for teaching and learning mathematics. We are creating a system that privileges the occurrence of mathematical situations, which places the student in a mathematical activity without the teacher's intervention (Brousseau, 2010, p. 21). The tasks constructed in these ATS are situated in didactic situations, thus,

"the teaching and teaching-learning processes to apply, require that the teacher provokes the student – through the sensible selection of the 'problems' they propose – adapting them to teacher intention. Such problems are chosen so the student can accept, make, using his own dynamics, that the student act, speak, reflect, and evolve" (Brousseau, 2010, pp. 34–35).

Along any didactic situation occurs an a-didactic situation that corresponds to certain moments of the learning process in which the student was not suffering any kind of direct control of the teacher about the mathematical content (Freitas, 2008, p. 84). In this way, a mathematical learning task in an ATS corresponds, in the first phase, to the didactic situation; in the second phase, the didactic situation made will trigger a set of a-didactic situations, ultimately perhaps in greater numbers than in face-to-face teaching.

Another relevant issue in the tasks designed in an ATS is to understand the extent to which they allow the self-regulation of student learning, which according to Leonor Santos and Paulo Dias (2013), "is the ability of the student to evaluate the execution of a task and make corrections, when necessary". In ATS, within each task, the feedback that can be given to each user's interactions corresponds to a strategy of self-regulation of student learning. However, the effectiveness of feedback depends on its opportunity and relevance. Regarding the principles of good feedback practices, Nicol & Macfarlane-Dick (2006) identify seven principles of good practice: i) help clarify good performance (goals, criteria, expected standards); ii) facilitate the development of self-assessment (reflection) in learning; (iii) provide students with high-quality information about their learning; vi) encourage dialogue between teachers and peers around learning; v) encourage positive motivational beliefs and self-esteem; vi) offer opportunities to reduce the distance between current and desired performance; vii) provide information to teachers that can be used to help shape teaching (feedback). These principles seem to us to be adopted when we think of a type of feedback that goes beyond the feedback of the mere certification of students' responses.

There are studies that have focused on the use of automatic feedback associated with presenting user scores in the correct responses and resolution times and in the sense that students can conduct self-assessment processes, particularly in problem-solving (Drijvers, 2018; Barana, et al., 2022). Recent studies have used GeoGebra's automatic demonstration capabilities to promote activities with automatic feedback on geometric and algebraic geometry problems (Kovács et al., 2020; Kovács et al., 2022). Issues related to the creation of automatic feedback in teacher training have also been addressed (Hašek, 2022). In the case of the project "GeoGebra as a strategy for remote teaching: creating activities with automatic feedback" (Dos Santos et al., 2022), the design of the tasks intended to provide feedback from a detailed analysis of a set of previously listed plausible student resolution strategies, with the teacher being the key to creating the appropriate feedback for the user task progress and, in a way, overcome the difficulties eventually reviewing or seizing concepts necessary for the challenge posed.

MATHEMATICS AUTOMATIC FEEDBACK TASKS

Tasks with automatic feedback can be defined as micro tutorial systems and could be integrated into an ATS. In general, the design of these tasks is quite sensitive, mainly if traditional techniques are used for systems based on rules created by experts (Inventado et al., 2017). The process of learning tasks design in face-to-face teaching is complex, but it relies on the interaction between the teacher and students. Hence, the communication established in the classroom allows the adjustment of the task and adaptation to students. Generally, in ATS, the interaction between students and the system cannot be monitored in the same way as in face-to-face teaching, which entails new challenges for the teacher or creator in these tasks. Let's review how this can be done, with some advantages, in the ATS we have been discussing.

Automatic Feedback Tasks in GeoGebra

Considering the automatic feedback tasks in GeoGebra, the feedback must be designed by an expert who is familiar with the mathematical contents, both from the didactic side as with the technological side. In particular, the technological domain of GeoGebra is fundamental to the design of these tasks. In addition to the correct answer, feedback could be provided by text images or sounds and incorporate other sub-tasks that allow the user to overcome the difficulties encountered in the face of an incorrect answer. One of the characteristics of tasks created with automatic feedback in GeoGebra is that the parameters can be randomly changed, so each task has an associated number of utterances that can vary in each user access.

An example of a task with automatic feedback, built with GeoGebra, of problems with words for the addition of natural numbers in the 1st year of schooling is shown in Figure 1.

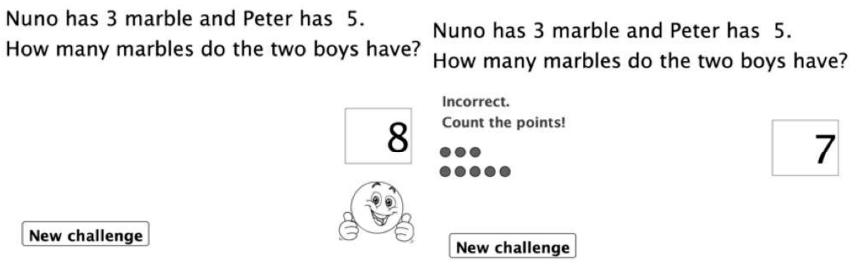


Figure 1: Word problems and adding integers in (Dos Santos et al., 2022, p. 79).

The option of the teacher was to give feedback with a static image in case of a right or wrong answer. In the case of the wrong answer, the feedback strategy is also visual, but in this case, it is dynamic because it depends on the utterance that changes each time the user presses the new challenge button. It should be noted that this feedback is based on the didactics inherent in adding natural numbers in the first year of schooling, first supported by the image students can use subitizing, and if they are unable to do so, they can count the points (Benoit et al., 2004), although this is a strategy to be gradually abandoned.

For each task, the creator, as an expert in content and didactics, builds an analysis matrix of the task, where he anticipates the possible answers of the students and elaborates feedback according to their didactic knowledge and the response given by the user (see the left side of Figure 2). At a later stage, a process diagram is drawn up that conducts the application's design in GeoGebra, in addition to schematising the actions to be taken in the face of the user's response (see the right side of Figure 2).

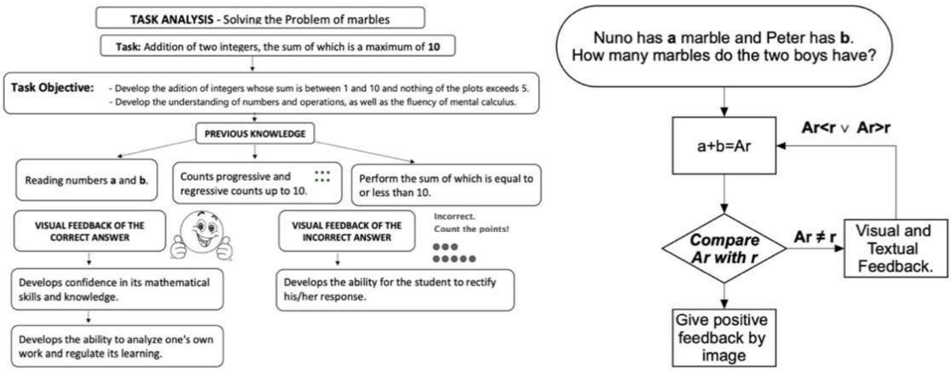


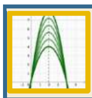
Figure 2: Matrix and Process Diagram of the task Word problems and adding integers in (Dos Santos et al., 2022, p. 80).

Although this example does not have subtasks, in the project "GeoGebra as a strategy for remote teaching: creating activities with automatic feedback", other activities were created where subtasks were used. Different types of inputs were used; the user, in addition to filling in text boxes, could fill tables; represent points and lines graphically. These actions are verified by the application and given the respective feedback.

Automatic Feedback Tasks in Asymptote

In the case of the Asymptote platform, in the 2022 version, the tasks have associated: a static image, a question whose answer may be multiple choice, a range, or a short answer. The task response or a set of clues, limited to three, can be given by text or a static image. Within each task, the user may have a maximum number of successes related to the number of clues given. By associating a set of tasks listed by a learning path previously designed by the task builder, we can establish a hierarchical order between tasks; some are called support tasks, and others are improvement tasks. The success or failure of the user in the learning path is that it determines the need to access the support tasks or be able to move forward to the challenge tasks. An example of a task with automatic feedback built-in Asymptote is Figure 3, which presents the first task built by one of the authors of this paper.

During the application design process, a first stalemate arose; the author of this task intended the image to be in GIF format, thus highlighting the dynamism of the task, whose response corresponded to a range of real numbers. However, the image could only be static. Other details were followed that have been overcome about the use of Latex and the natural learning process that the Asymptote platform requires. Later, in the process of submission of the task, it was necessary to change its categorisation to reasoning task, at the suggestion of the review process, a situation that led the author to change his strategy about the activities to begin to build in the portal for the construction of the learning path, schematized in Figure 4.



	Families of Parabolas Main tasks Task category: Reasoning
Task	The image depicts a family of parabolas passing through coordinate points (0.0) and (4.0). Consider point A, which belongs to one of the elements of this family, with abscissa 2 varying its order in the interval [4,8]. Knowing that the equations of this family can be described by equation y=α(x-0)(x-4), with α∈R, what is the range of variation of the α?
Answer	**[-2 , -1]**
Sample solution	Replacing x by 2 in the parabola family equation, we will have y=-4α. The vertex of the parabola family corresponds to point A, by which 4<-4α<8. Dividing inequalities by -4, we get that -2<α<-1.That is α∈[-2,-1]
Stepped Hints	1 Note that point A corresponds to the vertex of the family parables 2 Calculate the generic value of the abscess of A using the parabola equation. It is impossible to see that the expression of the abscissa of A framed between 4 and 8 allows obtaining the values of α. 3 You have to solve two inequations, 4<-4α and -4α <8. It observes that multiplying or dividing both members of an inequality by a negative number changes the signs of inequalities.
Explanation	Replacing x by 2 in the parabola family equation, we will have y=-4α. The vertex of the parabola family corresponds to point A, by which 4<-4α<8. Dividing inequalities by -4, we get that -2<α<-1. That is α∈[-2,-1].

Figure 3: Summary of the task "Parabolas family" available at https://www.asymptote-project.eu/en/portal-en/#!/task/t181992.

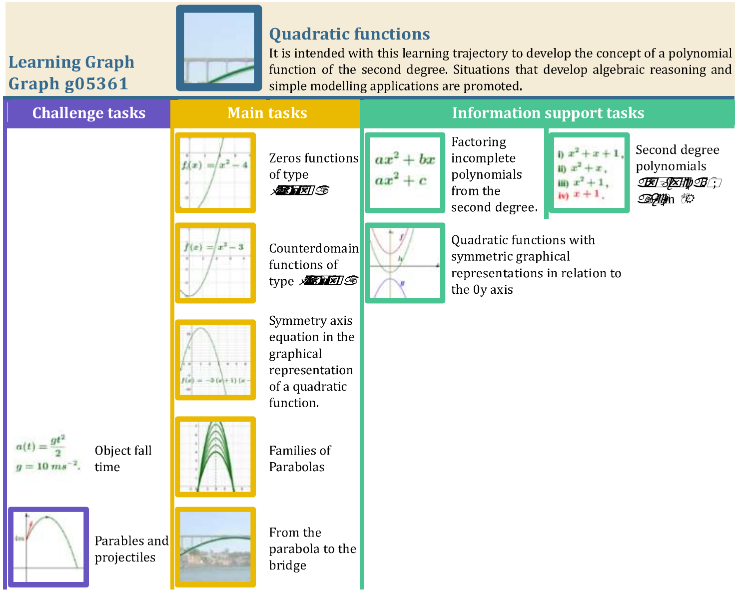


Figure 4: Tasks in learning graph "Quadratic functions" available at https://www.asymptote-project.eu/en/portal-en/#!/graph/g05361.

The learning path that came to be defined by the author involved in the construction of the previous task began to include five main tasks, three informational support tasks, and two challenge tasks.

In the learning graph, tasks were designed to have several ways to answer as fill-in, multi-choice, and value-indicating responses. Support tasks never included a specific user action other than reading the question, which included information and a series of questions for the user to think about the answers without them being directly asked for answers. The author understood that support tasks did not require specific user actions. Later the author came to know that this was different from the understanding of many other users of the platform because, in general, they put the student in need to interact with the platform, answering the support task.

VIEWS OF FUTURE TEACHERS AND STUDENTS

Tasks with automatic feedback in GeoGebra have been implemented in several classes of the 1st cycle of basic education in Portugal, these are prepared by the teachers, with interesting results, namely the one presented in Figure 2. These kinds of applications promote the autonomous work of students, with improvements in performance, being the technological context motivating and facilitating students, who interpret the work as a game (Alves et al., 2022). The teachers involved in the construction of these tasks considered it necessary to improve their technological knowledge, but at the same time, they mentioned that their investment was very positive in view of the results obtained.

Regarding the task and Learning Graph of the Asymptote platform presented here, it has not yet been tested with many students. In the review process, all tasks were accessed and made public. To perform a validation of the tasks and the learning graph, it was requested that two students in the last year of secondary school, Group I, and six students in a master's course on the teaching of mathematics in the third cycle of primary and secondary education, Group II, collaborate. Two virtual classes were created, in which the students performed the Learning Graph tasks and were asked to present their comments and opinions later.

Related to Group I, the two secondary school students considered that: “the Asymptote app is interesting and intuitive; when we work on the tasks seemed to be like playing a game; so, it became more fun to solve exercises”. One of the students also points out

"It is very good to be able to perform exercises with the essential challenges to learn the selected subject, but at the same time to be able to learn a little more about subjects that would be given later with the extra challenges."

There was only one suggestion for improvement in mathematical writing "in our mobile phone the square roots did not cover the numbers". It should be noted that during the tasks the students used paper and pencils to develop strategies and calculations and in addition, they established mathematical dialogue about the tasks.

The six students in Group II, future mathematics teachers, considered the tasks interesting and challenging. Three of them mentioned that they would consider using the Asymptote platform in practical classes with their future students. All of them pointed out that the platform seemed interesting to use, because it allows the creation of a kind of quiz, illustrates the functions, and not imposing a restricted time for students to perform the tasks. Regarding the learning graph tasks, all mentioned that the clues were useful in the quadratic part and in recalling certain properties of the parabolas. One of the students also mentioned: “I found the tasks interesting and challenging, covering several areas: quadratic functions (analysis) and projectile launches (physics)”. Like what happened in Group I, during the tasks, group II students also used paper and pencils to develop strategies and calculations: in this regard, one of the students states: "I felt that the tasks throughout the questionnaire were slightly increasing the degree of difficulty. It will be easier through a draft to come up with solutions by doing our auxiliary sketches and calculations."

Although less frequent than in the case of the dyad of Group I, some discussions about the tasks were also established in the dyads of Group II.

DISCUSION AND FINAL REMARKS

On both platforms, feedback is provided to the user; however, the opportunity and quality of the feedback are always related to the hypothetical learning trajectory (Simon, 1995) built by the teacher, essential for creating an automatic tutorial microsystem, which makes the student's learning path more effective. Thus, the feedback provided in the task, the hints or option to provide other additional tasks, is always guided by a hypothetical learning trajectory, where the teacher or programmer of the tutorial microsystem equates the thought and learning of students in the activity with feedback that creates. In the case of the face-to-face relationship in the classroom, Lurdes Serrazina and Isolina Oliveira (2010) that hypothetical learning trajectory guides teaching for the understanding of students or users of the application. Also, in the case of the Asymptote platform and GeoGebra's tasks with automatic feedback, the hypothetical learning trajectory previously defined is fundamental for creating tasks to get opportune and good feedback. Also, in the learning graph built on the Asymptote platform, presented here, the hypothetical learning trajectory was essential to achieve the objectives proposed a priori.

In the case of the automatic feedback tasks built with GeoGebra in the projects coordinated by Abar et al. (2022), the automatic feedback tasks were created according to the seven principles enunciated by Nicol and Macfarlane-Dick (2006), Each task included "a series of decisions by the author about pedagogical and curricular strategies inherent to the proposal, and also about the programming strategies in GeoGebra that they would use" (Abar et al., 2023, p. 24).

Although the number of participants is very small, the results reveal that the learning graph was adjusted, provoked interest in the students, and there is evidence to have triggered a-didactic situations in a sense defined by Brousseau (2010). It should be noted that Group II participants already knew and had studied the mathematical contents of the learning graph, but their records and the discussions they established allow us to affirm that students were involved in mathematical situations.

When we analyse the task production methods and learning paths in both platforms, we learn that the environments are engaging for users, largely by the fast way they have feedback, which in somehow makes the tasks look like a game. Clearly, in both cases, the key is who creates the content instead of a teacher. The process of creating content on both platforms will depend on the contribution of users, posing enormous challenges in the evaluation processes of these materials.

Considering the preparation of the Learning graph is simplest in the Asymptote platform since the user only must relate to previously elaborated tasks. However, tasks with automatic feedback in GeoGebra can be more user-effective, providing a wider number of mathematics representations and different ways of user interaction, but requiring much more domain technology by the content creator.

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