SOME REMARKS ON 'GOOD' TASKS IN MATHEMATICAL OUTDOOR ACTIVITIES

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Abstract. *Starting with a brief theoretical discussion of academic tasks, I shall point out two desiderata of task design in mathematical outdoor activities and subsequently propose a four-point scheme for assessing such tasks. I shall conclude with* three illustrative examples.

Key words: task design, outdoor mathematics, real world contexts

SETTING THE SCENE

Mathematical knowledge in an educational context is often organised along sets of tasks[1]. According to Lenné (1969, p. 50-54) this concept has its origin in the institutionalisation of schooling which has become part of the bureaucratic apparatus of modern societies. What is more, this very idea can be traced back to the training of bureaucratic elites in ancient societies. Yet – as I will argue – it is still reverberating in today's mathematics classrooms as well as in current research on mathematics education. Thus, it might be worthwhile to recall Lenné's description of 'task didactics':

> Each subdomain is determined by a specific type of task, which is to be dealt with systematically proceeding from simple to sophisticated patterns. Sophisticated tasks can be construed as combinations of simple tasks. Intrinsically, individual domains, therefore, appear to be rigorously systemised. Without being linked to each other, though, they are dealt with in a relatively isolated manner. 'Application tasks' are *separately* allocated to each domain. To make sure that one domain provides the required pre-conditions for the next, only the sequence of domains is determined. Domains which have been covered are considered as done, the pertinent subject-matter is taken for granted; interconnections along comprehensive ideas or structures are hardly worked out clearly – at least not systematically. In any case it applies: 'we have dealt with it' or 'we have not yet dealt with it'. Therefore, students do not meet mathematics as a whole intrinsically ideal entity, but as a mere repository of different types of tasks. I shall call this concept of how to organise subject-matter in traditional mathematics 'task didactics'. (Lenné 1969, p. 34-35)

It goes without saying that, from an educational point of view, this attitude towards mathematics leaves us wanting. Fortunately, however, there is no lack of alternative concepts, the *locus classicus* concerning tasks in the context of primary and especially secondary education being Doyle (1983).[2]

Doyle distinguishes four general categories of tasks: (i) memory tasks, (ii) procedural or routine tasks, (iii) comprehension or understanding tasks, and (iv) opinion tasks. This scheme has been widely accepted, and the differentiation of procedural and understanding tasks in particular has been stimulating research in education ever since. Its educational implications, however, are not yet fully realised. In theory, schools are thought to aim at comprehension and understanding, i.e. higher-level cognitive processes, in order to foster cognitive (and linguistic) development. In practice, though, schools all too often fail to achieve this objective. According to Doyle this could be explained by students' efforts to avoid both ambiguity and risk. In other words, in a school setting of constant monitoring and continuous accountability, students are not only striving for the security of tasks with

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low ambiguity and risk, but are actively avoiding or 'mis-performing' the didactically desirable, but highly ambiguous and risky understanding tasks (Doyle & Carter 1984, p. 144-147).[3]

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As a consequence, a central target of mathematics education is to design didactically desirable tasks,[4] withal empowering students to overcome the related structural hindrances of performance. In this case, an outdoor-setting[5] may provide the necessary and favourable environment to meet these challenges, especially when digital devices can be employed in addition. My arguments will be organised along the theoretical lines of Borba et al. (2013).

Collaboration

Working in small groups without direct monitoring by the teacher in an out-of-school environment can prompt self-determined and self-responsible action – but this requires a careful didactical framing. There has been extensive research on this topic, its very essence being summarised in three general, but powerful rules: Outdoor activities have to be (i) integrated into math classes with regard to content, (ii) prepared thoroughly, and (iii) individually revised afterwards (cf. Klaes, 2008).

Multimodality

To encounter real, i.e. physical, objects in an out-of-school environment and to regard them in the perspective of school-learning necessitates the active construction of interconnections between different domains of knowledge with the aim of fostering higher-level cognitive processes. This is especially true if the task on hand requires a thorough examination of the object. With access to the internet, information in the form of text and pictures can be easily retrieved to complement the outdoor learning environment.

Performance

Working on tasks in an out-of-school environment may contribute to deepen the very notion of mathematical knowledge. Mathematics would then be "no longer just the property of teachers and textbooks, nor [… be] constrained by the communication forms of traditional textbooks." (Borba et al. 2013, p. 692) Instead, it can be 'performed' outside the formal classroom setting, encouraging thereby an awareness of self-efficacy which may help in counteracting the well-known marginalising force of mathematical knowledge (cf. Sriraman et al., 2010).

REAL WORLD TASKS

As pointed out above, the didactically desirable aspects of mathematical tasks are intimately linked with real-world settings. To encounter real world objects and perceive them in a mathematically informed way, i.e. (mathematical) modelling, however, seems to be fraught with several difficulties. Two of these seem to me of particular importance.

First, mathematical modelling entails the competent handling of data, i.e. statistical literacy.[6] According to Gal (2002, p. 4) this encompasses knowledge elements (literacy skills, statistical knowledge, mathematical knowledge, context knowledge and critical questions) as well as dispositional elements (beliefs & attitudes and a critical stance).

Notably, focussing too much on mathematical (and at best statistical)[7] knowledge, the crucial role of context knowledge in accomplishing outdoor tasks is all too often underrated. A case in point of this *déformation professionnelle* is the bad habit of supplying the (allegedly) required context knowledge within the task itself – in striking contrast to the didactical considerations above.

Second, the very aim of mathematical modelling is insight and control, or in the famous words of Hertz:

> The most direct, and in a sense the most important, problem which our conscious knowledge of nature should enable us to solve is the anticipation of future events, so that we may arrange our present affairs in accordance with such anticipation. […] In endeavouring thus to draw inferences as to the future from the past, we always adopt the following process. We form for ourselves images or symbols of external objects; and the form which we give them is such that the necessary consequents of the images in thought are always the images of the necessary consequents in nature of the things pictured. […] When from our accumulated previous experience we have once succeeded in deducing images of the desired nature, we can then in a short time develop by means of them, as by means of models, the consequences which in the external world only arise in a comparatively long time, or as the result of our own interposition. We are thus enabled to be in advance of the facts, and to decide as to present affairs in accordance with the insight so obtained. (Hertz, 1899, p. 1)

Therefore, an outdoor task should allow for substantial insights into the object of concern to afford an adequate notion of mathematical modelling.[8] Expressed somewhat pointedly: Pay attention to the real world!

'GOOD' TASKS IN MATHEMATICAL OUTDOOR ACTIVITIES

As recapitulation of my above line of argumentation, I shall suggest a four-point scheme for assessing out-of-school tasks:

A 'good' task in mathematical outdoor activities has to meet four conditions: (i) students can gain substantial knowledge about real-world objects; (ii) students resort to substantial (school) mathematical knowledge; (iii) mathematics contributes to the understanding of the real-world context; (iv) a thorough exploration of the object of concern forms the core of activities.

To illustrate the above criteria, I shall discuss a generic example from the research literature and suggest two examples from my own teaching project.

Example 1: Art Gallery

I shall begin with a task by Buchholtz (2017), re-reading it as a generic illustration of a blind spot widespread among mathematics educators, acknowledging that it has been devised in a diagnostic setting, albeit – in the words of Buchholtz – 'based on meaningful reality-based tasks'.

The example, chosen from a mathematical city walk, is the problem called 'Art Gallery', which deals with the ramps (and stairs) leading up to the forecourt of *Kunsthalle Hamburg (Hamburg Art Gallery)*. It reads as follows:

> (i) The platform [i.e. the forecourt] can be accessed via four long ramps, which are divided by the stairs into a left and a right section. Compare the two sections of the ramp with each other. What do you notice?
>
> (ii) At the stairs one can measure what height one section of the ramp overcomes [sic]. Record the measured height in the worksheet.
>
> (iii) Also wheelchair users would like to reach the platform. Wheelchair ramps may however, for safety reasons, only have a slope of no more than 6%. Find out how long a section of the ramp is. How can you determine the slope in percent (%)? Can wheelchair users navigate safely on the ramp?

In his book chapter, Buchholtz discusses the task mainly in regard to its diagnostic potential concerning the calculation of percentage and of slope. The real-world context in the form of wheelchair users is mentioned only once: "If the sizes were determined correctly, the result is a slope of about 6.5% […]. With regards [sic] to the problem for wheelchairs, it can be stated that driving onto the ramps would not be safe […], although […] certainly still possible." (cf. Buchholtz, 2017, p. 53 f.)

With regard to the real-world context, this passage falls way too short. To begin with, there is no discussion of the accuracy of measurement (except for measuring a false quantity). Furthermore, there is neither a discussion of the accuracy required in the construction of ramps (which coincidentally amounts to 0.5%) nor a discussion of further specifications of wheelchair ramps. A short glance in the relevant (German) regulation, DIN 18040-1, reveals numerous shortcomings: The mandatory rest platform for every 6 m of length is missing (the ramp at *Kunsthalle Hamburg* runs for over 11 m without a rest platform) just as the prescribed hand and wheel rails. Going beyond national regulations and comparing them, for example, with the corresponding regulations in the USA, the *Americans with Disabilities Act (ADA)*, one finds a maximum permitted slope of 1 : 12 (one foot ramp for one inch of rise), thus exceeding 6% by far.

To sum up: the reference to wheelchairs in the third subtask seems dispensable in terms of diagnostics, while in terms of mathematising real-world contexts, it is a missed opportunity of learning something substantial about wheelchair ramps. Of course, it may be granted that the task is primarily about slope and not about wheelchair ramps – but this is exactly what I mean by saying that 'task didactics' is still reverberating in current research.

Example 2: Humboldt Penguins

The following task originates from my current teaching project *Out-of-School Learning in Frankfurt*. For two years now, every semester I have chosen an out-of-school place in Frankfurt, visiting it weekly with ten students to find situations containing mathematics and trying to design 'good' math tasks. In *Frankfurt Zoo*, a student proposed the following problem for pupils of age 10-12 (due to place constraints, I shall shorten the text):

(i) How many Humboldt penguins are living in *Frankfurt Zoo*? First, count on your own, then compare your results among each other. Did you all get the same result? (ii) Count the penguins methodically, ensuring no penguin is overlooked or counted twice. Describe how you went about the task. (iii) How many penguins are male, how many female? (Hint: Take a look at the name badges.) (iv) Look at the penguins more closely. Can you differentiate males and females without looking at the name badges? If so, how? (v) Observe one penguin of your choice for ten minutes while s/he is diving. How often does s/he submerge and for how long does s/he stay under water each time? Record your data in a table. (vi) Present your data in the form of a box plot.

In preparation for the excursion to the zoo, the pupils are asked to catch up on Humboldt penguins on the internet, and afterwards are assigned to find out about the biological characteristics allowing penguins – a species of a bird (!) – to dive.

This task allows pupils to gain knowledge about Humboldt penguins, by the way a vulnerable species (*inter alia* due to climate change): Their sexual dimorphism, for example, is very subtle, and only very thorough examination by experts allows for differentiating between males and females. Information (suitable for various age classes) about their adaption to aquatic life is readily and abundantly found online. Mathematically, not only the intricacies of counting and measuring are addressed, but also the value of repeated and systematic observations to gain insight in the diving behaviour of Humboldt penguins beyond a trite 'can stay under water up to … minutes' – even with the prospects of discussing behaviour in captivity.

Example 3: Macrotermes spp.

The following problem originates from the same project, this time visiting *Senckenberg Museum*, a museum of natural history in Frankfurt. The (shortened) task of my own design reads as follows:

Figure 1: A showcase at *Senckenberg Museum* displaying a termite mound of *macrotermes spp.* on a scale of 1 : 1 (picture taken by the author).

(i) Find the model of the termite mound of *macrotermes spp.* [cf. Fig. 1] and read the information panel (*spp.* standing for: varying species of the genus *macrotermes*). (ii) Higher termites live in highly organized colonies often exceeding over a million individuals. Usually there are three castes: the reproductive caste (as a rule one queen and one king), the soldier caste and the worker caste. Inform yourself about eusociality. (iii) How many times larger is the queen in comparison to a worker? Refer to the models of *macrotermes spp.* in the display next to the mount [cf. Fig. 2]. (iv) If proportions in the world of humans and termites would be the same, what would be the size of a human 'queen'? (v) Determine the height of the termite mound as accurately as possible. Again, if proportions in the world of humans and termites would be the same, how high would be a human 'mound'? Compare your result with the height of the Frankfurt Trade Fair Tower.

Further subtasks cover topics such as biological systematics, hemimetabolism and physiogastrism.

As an addendum, there is an excerpt from Smeathman (1781), a letter written to the *Royal Society of London* concerning observations on termites, containing a footnote outlining a line of thought similar to that in subtask (v):

> The labourers are not quite a quarter of an inch in length; however, for the sake of avoiding fractions, and of comparing them and their buildings with those of mankind more easily, I estimate their length or height so much, and the human standard of length or height, also to avoid fractions, at six feet, which is likewise above the height of men. If then one labourer is = to one-fourth of an inch = to six feet, four labourers are = to one inch in height = 24 feet, which multiplied by 12 inches, gives the comparative height of a foot of their building = 288 feet of the building of men, which multiplied by 10 feet, the supposed average height of one of their nests is = 2880 of our feet, which is 240 feet more than half a mile, or near five times the height of the great pyramid; and, as it is proportionably wide at the base, a great many times its solid contents. If to this comparison we join that of the time in which the different buildings are erected, and consider the Termites as raising theirs in the course of three or four years, the immensity of their works sets the boasted magnitude of the ancient wonders of the world in a most diminutive point of view, and gives a specimen of industry and enterprize as much beyond the pride and ambition of men as St. Paul's Cathedral exceeds an Indian hut. (Smeathman, 1781, p. 148 f.)

This task tries to stimulate an 'in-depth' exploration of (higher) termites along the exhibits of *Senckenberg Museum*. As social insects, termites provide a wealth of features that have fascinated mankind for centuries, notably their highly developed construction activity (parental care would be another feasible topic). The measuring and reckoning approach taken – mathematically a mere exercise in elementary proportions – was perceived favourably by my students as a way of coming to terms with the wondrous 'industry and enterprize' Smeathman so aptly describes.

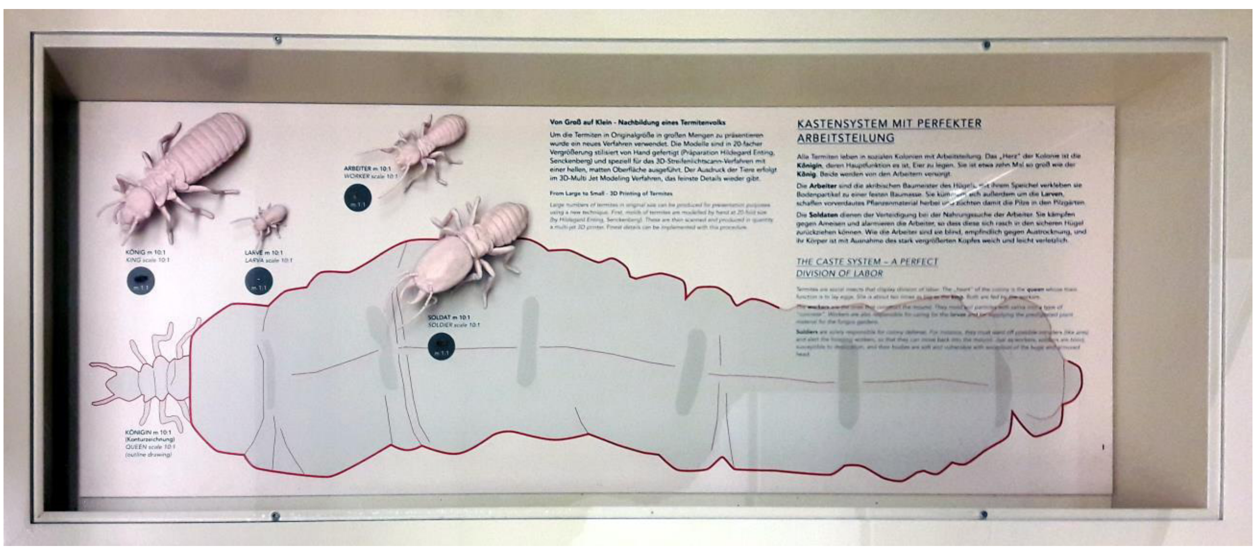

Figure 2: A showcase at *Senckenberg Museum* displaying ideal-typical individuals of different castes of *macrotermes spp.* on a scale of 10:1 (picture taken by the author).

EPILOGUE

In both of my own examples, the real-world context comes first. In fact, at the beginning several students of mine felt the mathematics involved would not be sufficient. While it is true that counting, measuring, and proportions are quite elementary, it is equally true that they form a substantial part of the core of mathematical reasoning – a point neglected on occasion. Many of the reservations, it seems to me, are voiced mainly due to unfamiliarity. My students, as a case in point, came forward with beautiful ideas rather quickly, and I can see no reason, why teachers with all their experience and expertise should not be able to do the same, provided they dare to transgress the line of 'proper school mathematics' without the sense of guilt passed on by tradition and fueled – as I have tried to argue – by the tradition of 'task didactics'.

Admittedly, teachers looking for a set of 'application tasks' (to take up Lenné's term) in a specific domain of school mathematics would perhaps not find anything worthwhile in the kind of examples I propose. I would assert, nevertheless, that there are already too many examples of 'application tasks' of high quality to add but another one; and I would add for consideration, that there is a price to pay for trading school mathematical concerns above value: a quite twisted image of what (school) mathematics is all about.

If you are bent on doing (school) mathematics, the classroom seems the place to be. But when you are out for a genuine engagement in outdoor activities, let your maxim be: real world first, mathematics second.[9]

Notes

1 In specialised contexts such as problem-solving, it is common practice to differentiate between tasks and problems; in the following, I shall use these words synonymously.

2 For the adaption of Doyle's work to mathematics education cf. Stein et al. (1996).

3 In mathematics education, instead of 'understanding tasks' a variety of terms are in use: 'cognitively demanding tasks', 'rich tasks', 'powerful tasks' or 'substantial tasks', to name but a few.

4 Task design has never been at the core of research in mathematics education. Nevertheless, the last decade has provided some relevant literature, notably Sullivan et al. (2013) and Watson et al. (2015). For the German discourse, the pertinent reference is Leuders (2015).

5 In the following, I shall extend the notion of outdoor activities to include any out-of-school activity.

6 Astonishingly, this obvious fact is not worthy of note in nearly all educational concepts of mathematical modelling, a pertinent example being the well-known modelling circle of Blum et al. (Blum & Borromeo Ferri 2009, p. 46).

7 To name but the most salient aspect of statistical knowledge: Quantitative results are only meaningful as far as one has an idea of its bounds of error. In stark contrast to this commonplace, there is virtually never a trace of a rough guess about the accuracy of the results obtained in mathematical modelling tasks in educational settings.

8 Stillman et al. (2013, p. 2) suggest four dimensions of 'authentic' modelling: content authenticity, process authenticity, situation authenticity, and product authenticity. Arguably, not any kind of modelling is obliged to authenticity (cf. Kaiser & Sriraman 2006), especially in an educational context; but the respective arguments are not easily transferred to an out-of-school context.

9 It is my pleasure to cordially thank Gerhard Bierwirth and Lutz Führer for their invaluable advice once again.

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