EXPLORING REAL WORLD ENVIRONMENTS USING POTENTIAL OF GEOGEBRA AR

Bernat Ancochea[1] and Martha-Ivón Cárdenas[2, 3]
[1] Federación Española de Sociedades de Profesores de Matemáticas, Spain (FESPM),
[2] Federació d'Entitats per a l'Ensenyament de les Matemàtiques a Catalunya, Spain (FEEMCAT)
[3] Universitat Politècnica de Catalunya, Spain (UPC), Computer Science department

Abstract. *Two projects that we present in this paper are a proposal that allows students to open their eyes to the mathematical world around them, as if they were looking for a treasure of 2D or 3D geometric patterns that are waiting to be discovered by some curious person. Through the platform MathCityMap, a series of challenges will be found on the tour which include augmented reality that allow them to bring to life many objects of great cultural, historical or architectural importance. Attractive to any type of student, all the tours take mathematics out of the classroom and brings it closer to our everyday world through motivating and active learning, encouraging geometric thinking. In that sense, GeoGebra 3D with the augmented reality tool both facilitate the modelling of geometric real-world objects.*

Key words: GeoGebra 3D, GeoGebra AR, augmented reality, modelling.

INTRODUCTION

In this paper we report two projects aimed at middle school students and designed to explore and (re)discover the mathematical world. Intended to stimulate engagement and enthusiasm for maths through its connection with history, art or architecture contents, these projects facilitate integration and tailoring of multidisciplinary contents to the curricular and educational needs. The platform that we used here named MathCityMap (https://mathcitymap.eu) allows us to create the mathematical routes. The use of math routes has an extensive literature noted at (Jablonski et al., 2020; Richardson, 2004; Cahyono & Ludwig, 2019; Ludwig & Jablonski, 2019) and also many projects have been published in its repository. The novelty that we present here consists of modelling real-world architectural objects using GeoGebra 3D (https://www.geogebra.org) with its add-on augmented reality (AR) tool.

Goals

The goals of this paper are to experience mathematics easily outdoors, to promote geometric discovery and to connect with other subjects enhancing the science, technology, engineering, arts and mathematics (STEAM) learning through the use of digital manipulatives tools.

METHODOLOGY

Both projects are aimed at middle school students but it can be extended also to high school students by adapting the activities. They have in common to support the development of geometric thinking through the effectiveness of GeoGebra AR. For that purpose, curricular requirements are included in these projects which reinforce mathematical concepts from current or previous courses.

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All the students are organized in groups of 3-4 and each group interact in a collaborative way. Previously to the routes, they have to ensure that they have downloaded the GeoGebra 3D application in their mobiles or tablets, as well as one integrant of the group need to have the AR tool available on his device for the handling and adjustment of the geometrical models.

Concrete directions for a successful activity are given to students as follows:

1 To assign a role: a coordinator, a note/photo-taker and communicators, depending on the number of students.
2 To plan the route: the have to study the locations and strategies.
3 To identify and model geometric objects: explore the mathematical properties and try to solve the challenge.
4 To collect data and evidences of each locating point of the route: ensure to take notes, take a picture of the final product and connect math content with that learned in class.
5 To work collaboratively to create a presentation and a summarizing report.

Before the projects are approved, some teachers test the routes in order to adapt better the activities to the variety of students.

MathCityMap is an application that generates maps with activities and is multiplatform. It is a project of the University of Frankfurt in collaboration with other countries and universities.

GeoGebra is dynamic mathematics software for all levels of education that brings together geometry, algebra, spreadsheets, graphing, statistics and calculus in one easy-to-use package. GeoGebra has become the leading provider of dynamic mathematics software, supporting STEAM education and innovations in teaching and learning worldwide.

Augmented reality (AR) (Muñoz, 2020) allows the user to mix real images with virtual images using electronic devices. The main features of AR are that combines real and virtual elements, it is interactive in real time and it is made in 3D.

There are different types of visualization:

Level 0. This is the lowest level and offers the minimum interaction with the physical world. The applications link the physical world with the virtual world by using barcodes and 2D codes (for example, QR codes).

Level 1. Applications use markers, 2D patterns that trigger the appearance of three-dimensional objects on them.

Level 2. Applications use images, objects or GPS coordinates to overlay virtual information.

Level 3. This is the highest level and would be covered by devices such as glasses or contact lenses that project information on what is being seen in real time.

GeoGebra performs an AR **level 3** through the mobile phone or the tablet by mixing real image with virtual image without the need of any marker.

Didactical competences and skills applied in the projects

- Mathematical competence: concepts used include: formulas, data, variables, three-dimensional coordinates and basic mathematical operations and statistics, reading and interpreting graphs, geometry and perspective, areas and volumes.
- Communicative competence: through a technical and communicative language the students will have to write reports and documents, write personal opinions, write texts of different typologies, exhibition and communication techniques, oral presentations, audiovisual and multimedia resources.

- Information processing and digital competence: by means of the new computer tools of the information and communication technologies linked to the development of the technical projects in all their phases. Students will acquire skills with graphic and three-dimensional design programs as well as video editing.
- Competence to learn how to learn: through the phases of the technological project they will have to obtain information, analyze it and select all the useful information related to their purpose to solve the challenge posed.
- Competence in knowledge and interaction in the physical world: this competence will be worked from the point of view of 3D modelling with augmented reality (volumes, faces, centroids, symmetry, proportionality, ratio aspect and rotations).
- Social and citizen competence: the designed routes are aimed at improving the cognitive, personal and social development of the students, as well as the impact of mobile learning in mathematics education.

Mathematical models applied in the projects

Theoretical foundations

- Recognize and classify polyhedrons and their elements.
- Develop the polyhedrons and obtain the surface and volume.
- Recognize, name and describe regular polyhedra.
- Solve geometrical problems involving the calculation of surfaces and volumes in bodies of revolution and polyhedra.

How do we mathematize the route?

- Referencing images: prior to the route and exploring geometric properties.
- Geolocating the tasks.
- Having a 3D surface repository.
- Generating didactic ideas.
- Using GeoGebra applets: create an ID and make it public.
- Giving augmented reality (AR) to the constructions.
- Creating new tools: it would simplify the constructions.

PROJECTS DESCRIPTION

The projects exposed here are designed to take place at the north of Barcelona in the region of Maresme coast which is rich in ruled geometry and historical architecture.

Project I: Mathematical walks at Maresme Sud

Our project would focus not only on geometrical objects founded around us, specially 3D as prisms, cones, cylinders, spheres and 2D examples of parallel and perpendicular lines, but also on probabilities, statistics, slopes and so on. We expect that the math walks will last one hour and a half. Our idea is to adapt the route and the questions after a first test walk and even later if pupils have new proposals because we consider that their contribution is basic in order to have a sort of *dynamic routes*.

It is headed by a group of teachers of Primary and Secondary schools at Maresme Sud, placed at the north of Barcelona that are registered in the Educational Resource Center (CRP) (https://serveiseducatius.xtec.cat/baixmaresme/categoria/crp). In coordination with that entity, teachers plan to prepare mathematical walks through different villages from Maresme Sud: El Masnou, Premià de Mar, Premià de Dalt, Vilassar de Dalt and Argentona. The starting point was a mathematical walk tested at the Mercat de Sant Antoni, Barcelona and surrounding objects (see figures 1 and 2). Departing from a GeoGebra construction previously done, students need to explore the model surfaces in parts or in total, using rotations, scaling and superposing the model with AR. Another very interesting proposed activity involved in the project is the research of different kind of surfaces: ruled surfaces, arched surfaces, surfaces of revolution. It is very easy to find many of them in many places, not only in churches or town councils but also in historical buildings. In that sense, GeoGebra is a very useful technique that allows us to draw a variety of surfaces for which there is no need to design complex mathematical models. Also, the planned activities include to add to GeoGebra some extra tools which can be helpful to design surfaces in a little number of steps.

Looking for symmetries, rotations, translations or other transformations is a very useful aim in a mathematical walk. But in addition, we suggest pupils to discover them in buildings or in nature. Studying art subjects contributes to the development of essential skills as it is well known and we will include some of them in the walks.

Figure 1: Examples of a real object modelled with GeoGebra 3D.

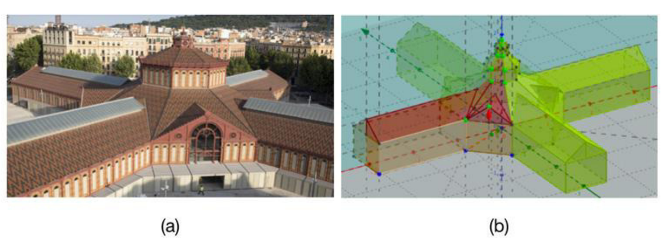

Figure 2: Examples of real object to model with GeoGebra 3D.

Project II: Mathematical walk exploring modernist fronts in Sant Pol de Mar with GeoGebra AR

This project invites students to enjoy a mathematical walk where the protagonists are the modernist fronts that, together with other geometric elements, allow to explore their symmetries, spatial distributions and to model some of their elements. It is located at Maresme Nord which covers the north area of Maresme and complement the geographical region of project I. Although there are only physical tools in MathCityMap platform like measuring tape, calculator or level, students also use digital tools included in GeoGebra as well as AR. The area occupied by the tour is 0.15 km^2 (see Fig. 3). Having a path with a total length of 0.8 km, the expected duration is about 2 hours and 40 minutes.

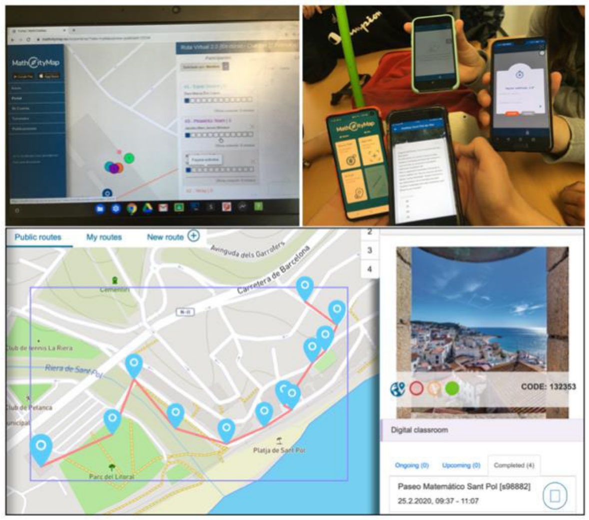

Figure 3: Images of the MathCityMap platform and the app installed in students' mobiles.

Figure 3 shows some images of the route. In the picture top-left can be observed a map with dots which shows students geolocated. In the picture top-right students prepare in groups the strategies for the walk. In the picture bottom-left is showed the path designed in MathCityMap platform and in the picture bottom-right is showed the representative picture of the all walk. The access code to do the walk from the platform is 1323552. Figure 4 shows a sample of the activity two, when students need to model a little house at a children park.

Evaluation of the project II

The project II was evaluated by the students through a Google form to measure the mathematical knowledge generated with the activities, data collection, analysis, interpretation and degree of satisfaction. A remarkable aspect of the survey is that 65% of the students answered in a positive way about the math learning contents improved. Summarizing, the 53% could solve the tasks in the planned time for the walk. The 49 % did not ask for any help. Finally, the 71% was agree that the math walk is interesting, the 12% was strongly agree whereas the 16% was undecided or disagree, which in general is a good result.

Figure 4: Activity 2 of the walk: (a) is the location and (b) is the GeoGebra model of one object from the Park.

CONCLUSIONS AND FUTURE WORK

In general, the objectives of both projects have been achieved enabling students to learn 3D geometry in a meaningful and efficient way. Project I which is still open, is evolving while adds a variety of models 2D and 3D to its collection that has been tested and are almost ready to apply in schools. This initiative involves schools from Maresme Sud that will create the future walks in a collaborative way. To accomplish that connection, MathCityMap platform will be useful while schools interact with each other. In project II, students found the experience very successful. However, some geometric models were very complex leading to a slow develop of the activity due to the limitation of the graphic card of their devices. Further, they came up with some suggestions for improve their routes and some students get encouraged to create their own activity and add it to the route. In the future, we are planning merge both projects and extend the mathematical routes around Catalunya by creating a large network with our own didactical stamp.

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